

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES MODELLING AND SIMULATION OF FULL-BRIDGE SERIES RESONANT CONVERTER BASED ON GENERALIZED STATE SPACE AVERAGING

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ABSTRACT

The resonant converters have advantages for high power or high frequency power conversion. The small signal modeling technique based on the generalized state space averaging method is applied to full-bridge series resonant DC/DC converter. According to the simulation, the open loop frequency characteristic curve is obtained from MATLAB, this paper analyzes the influence of duty ratio, input voltage to frequency characteristics, then build a close loop simulation circuit with MATLAB, the simulation results shows that, the small signal model has good controllability, generalized state space averaging method is more accurate to series resonant converter modeling. The equivalent circuit model can predict the dynamic behavior very well when switching frequency is below, close to or above resonant frequency.

Keywords: *Series Resonant Converter, Small-signal model, Generalized State-space averaging, MATLAB*

I. INTRODUCTION

The Switched Mode Power Supplies [SMPS], with their higher efficiency and reduced size, have almost totally replaced the traditional linear regulated power supplies. Significant progress in the fields of circuit topologies, semiconductor power devices, control theory and integrated circuits, have made the switch mode power supplies, a mature technology. However, the recent remarkable advances in Integrated Electronics, have greatly reduced the size of many electronic systems, and in order to fully utilize the advantages of denser electronics. It is well recognized that higher power densities are possible with higher switching frequencies. The voltage/current stresses on transistors and diodes can be minimized by means of the correct choice of transformer turn ratio [3-10]. The resonant converters have advantages for high power or high-frequency conversion [3-7]. This is due to their accommodating nature of parasites of HV transformer in their tank circuit, efficient power conversion, and generation of lower EMI. Two modulation techniques are usually employed. Instead of frequency modulation control, the resonant converters can also be regulated by phase-shift control[4-10], where the duty cycle is modulated and the switching frequency is kept constant. As most of the application involve a regulated voltage output, therefore a feedback loop is incorporated into the control system to stabilize the output voltage [2]. For power electronic circuit concerned, the commonly used small signal modeling method to get PWM converter dynamic model is state space averaging method, equivalent switching circuit method, the premise condition to apply the above methods for PWM circuit dynamic modeling is in one or more than one switch period, the state variable range is small compared with static in terms of volume. But resonance working circuit form part, the mentioned methods cannot be used. Because the resonant converter is rather complicated, in one or more switch cycle, each state variables is sine (cosine) term, or its effective ingredients is sine (cosine) term, amplitude ranges in large scale, so the use of “small ripple hypothesis” in the PWM converter is no longer applicable to the small signal modeling of the resonant tank. The harmonic current and harmonic voltage concerned can be considered as disturbances which is large size and whose dc average volume is small, thus the conventional linearization is not suitable for the form. So the other effective methods must be carried on the small signal modeling for the resonant circuit topology. The Generalized State Space Averaging method (GSSA) is a simple and effective method [1-3].

II. THE FULL BRIDGE SERIES RESONANCE CIRCUIT DYNAMIC MODELING

Generalized state space averaging method is based on the main idea with series resonant tank of the converter, including the resonant tank, switch network, rectifier and filter and other network, then write nonlinear state equation by KCL, KVL column, nonlinear links will be linearized by using generalized state space averaging method, and large signal static model will be obtained according to the fundamental harmonics approximation (FHA), based on this, the static model interference is changed and linearization, some static working point of small signal model obtained. This paper takes the full bridge series resonant circuit topology for example. The circuit form is shown in figure 1. The typical waveforms of the relevant state variables are shown in figure 2 for full bridge series resonant circuit when phase shift PWM control is used. As shown in figure 2, resonant tank input voltage (Vab) is square wave, resonant inductance current and resonant capacitance voltage is approximate sine wave. The dynamic mathematical model of the circuit is established by using the generalized state space averaging method [2]. According to a generalized state space average method, the resonant inductance current and resonant capacitance voltage are approximately written in the form of equation 1 & 2 [1], [2].

$$i(t) = i_s(t)\sin(\omega_s t) + i_c(t)\cos(\omega_s t) \tag{1}$$

$$v(t) = v_s(t)\sin(\omega_s t) + v_c(t)\cos(\omega_s t) \tag{2}$$

In equations 1 & 2, i_s, i_c represent resonant inductance current and v_s, v_c represent resonant capacitance voltage as state variables. Therefore, the circuit topology has five state variable namely i_s, i_c, v_s, v_c and v_o which is output voltage of filter capacitor. The full bridge series resonance circuit is a typical 5 order system [1], [2]. Differentiating equations 1 & 2, we get following equations.

$$\frac{di}{dt} = \left(\frac{di_s}{dt} - \omega_s i_c\right) \sin(\omega_s t) + \left(\frac{di_c}{dt} + \omega_s i_s\right) \cos(\omega_s t) \tag{3}$$

$$\frac{dv}{dt} = \left(\frac{dv_s}{dt} - \omega_s v_c\right) \sin(\omega_s t) + \left(\frac{dv_c}{dt} + \omega_s v_s\right) \cos(\omega_s t) \tag{4}$$

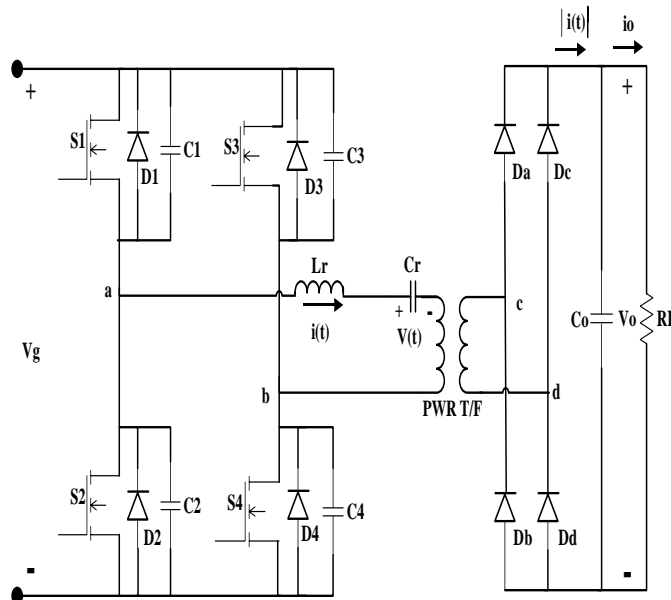


Figure 1: Full bridge series resonant circuit topology structure

The turn ratio of transformer is 1:1, for continuous tank current the list of equations of state variables are given below.

$$v_{ab} = L_r \frac{di}{dt} + v + \text{sgn}(i)v_o \tag{5}$$

$$i = C_r \frac{dv}{dt} \tag{6}$$

$$|i| = C_o \frac{dv}{dt} + \frac{v_o}{R_L} \tag{7}$$

Where $\text{sgn}(x)$ is signum function, that is:

$$\text{sgn}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \tag{8}$$

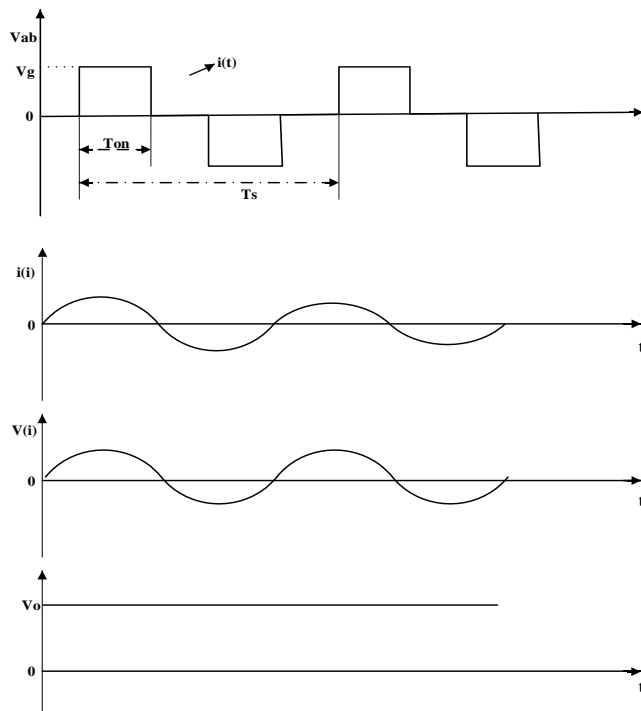


Figure 2: Typical waveform of state variables of SRC.

Using Fourier series expansion and base wave approximation voltage across the resonant tank, resonant current and output voltage are approximate and given below.

$$v_{ab}(t) = \frac{4v_g}{\pi} \sin(w_s t) \sin\left(\frac{\pi}{2}d\right) \tag{9}$$

$$\text{sgn}(i)v_o = \frac{4i_s}{\pi i_p} v_o \sin(w_s t) + \frac{4i_c}{\pi i_p} v_o \cos(w_s t) \tag{10}$$

$$|i| = \frac{2}{\pi} i_p \quad (11)$$

$$i_p = \sqrt{i_s^2 + i_c^2} \quad (12)$$

By substituting (3 &4) and (9 to 12) into (5 to 8) and by equating the coefficient of dc sine term and cosine term respectively, we get a 5 order equation. The first two equations (13 & 14) are coefficient of resonant inductor and equations 15 and 16 are coefficient of resonant capacitor voltage and 17 is output current equation.

$$L_r \frac{di_s}{dt} = L_r \omega_s i_c - v_s - \frac{4}{\pi} \frac{i_s}{i_p} v_o + \frac{4}{\pi} \sin\left(\frac{\pi}{2} d\right) v_s \quad (13)$$

$$L_r \frac{di_c}{dt} = -L_r \omega_s i_s - v_c - \frac{4}{\pi} \frac{i_c}{i_p} v_o \quad (14)$$

$$C_r \frac{dv_s}{dt} = C_r \omega_s v_c + i_s \quad (15)$$

$$C_r \frac{dv_c}{dt} = -C_r \omega_s v_s + i_c \quad (16)$$

$$C_o \frac{dv_o}{dt} = \frac{2}{\pi} i_p - \frac{v_o}{R_L} \quad (17)$$

We assume that all relevant variables are static and its stable value are $I_s, I_c, V_s, V_c, V_o, D, V_g, W_s$. The D is static duty ratio, W_s is static switching angular frequency [2]. The static value of I_s, I_c, V_s, V_c are given by:

$$I_s = \frac{4V_g}{\pi} \frac{R_{eq}}{R_{eq}^2 + X_{eq}^2}$$

$$I_c = \frac{4V_g}{\pi} \frac{X_{eq}}{R_{eq}^2 + X_{eq}^2}$$

$$V_s = \frac{4V_g}{\pi W_s C_r} \frac{X_{eq}}{R_{eq}^2 + X_{eq}^2}$$

$$V_c = \frac{4V_g}{\pi W_s C_r} \frac{R_{eq}}{R_{eq}^2 + X_{eq}^2}$$

$$X_{eq} = W_s L_r - \frac{1}{W_s C_r}, R_{eq} = \frac{8}{\pi^2} R_L \quad (18)$$

In fixed frequency and variable duty ratio (phase-shifting angle) only duty ratio is linearized and interference in generated and filter capacitor voltage is taken as output. In the process of the linearization and interference, to contain state variables of the nonlinear part, multiplied by reasonable Taylor series expansion and partial differential approximate, from equation sets (13 to 17) we get small signal equation of state matrix and output matrix given below.

$$A = \begin{bmatrix} \frac{4I_c^2 V_o}{\pi L_r I_p^3} & W_s + \frac{4I_s I_c V_o}{\pi L_r I_p^3} & -\frac{1}{L_r} & 0 & \frac{4I_s}{\pi L_r I_p} \\ \frac{4I_s I_c V_o}{\pi L_r I_p^3} - W_s & \frac{4I_s^2 V_o}{\pi L_r I_p^3} & 0 & -\frac{1}{L_r} & \frac{4I_s}{\pi L_r I_p} \\ \frac{1}{C_r} & 0 & 0 & W_s & 0 \\ 0 & \frac{1}{C_r} & -W_s & 0 & 0 \\ \frac{2I_s}{\pi C_o I_p} & \frac{2I_c}{\pi C_o I_p} & 0 & 0 & -\frac{1}{R_L C_o} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{2V_g \cos\left(\frac{\pi}{2}D\right)}{L_r} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [0 \ 0 \ 0 \ 0 \ 1]$$

$$D1 = 0 \tag{19}$$

From the above mentioned value of matrices A, B, C and D1, transfer function of small signal model of the output v_o to the control variable duty ratio d can be obtained by following relation.

$$G_{vd}(S) = C(sI - A)^{-1}B + D1 \tag{20}$$

III. SIMULATION RESULTS

In the previous section we have developed a small signal model of the resonant converter. This model is exemplified with illustrative calculation in this section for a converter whose parameters are listed in table 1.

Table 1: Parameters of converter for illustrative calculations

| Parameters | Values | Parameters | Values |
|---------------------------|-----------|--------------------------------|-------------|
| DC input | 560 | Duty ratio (D) | 0.2-0.9 |
| Transformer turns ratio | 1:1 | Output filter capacitance (Co) | 0.7 mH |
| Resonant frequency (fr) | 17.8 kHz. | Load resistance (RL) | 22 Ω |
| Switching frequency(fs) | 22 kHz. | Output DC voltage (Vo) | (340-550) V |
| Resonant inductance (Lr) | 100 μH | Output DC current (Io) | 20 A (max.) |
| Resonant capacitance (Cr) | 0.8 μF | Voltage divider ratio | 1:73 |

MATLAB is used for the calculation of the transfer function with the parameter given in table 1 and the following steps

- Calculate A
- Calculate B
- Calculate C
- Calculate D_I

Simulation of open-loop system

The open loop transfer function of series resonant power circuit (G_{vd}) of prototype SMPS is calculated using equation (20) which is given below.

$$G_{vd}(s) = \frac{6.95 \times 10^9 s^3 + 6.59 \times 10^{14} s^2 + 2.19 \times 10^{19} s + 1.58 \times 10^{25}}{s^5 + 1.32 \times 10^5 s^4 + 6.32 \times 10^{10} s^3 + 4.17 \times 10^{15} s^2 + 4.4 \times 10^{19} s + 3.2 \times 10^{22}} \quad (21)$$

Figure 3 is the frequency response (Bode plot) of open loop transfer function $G_{vd}(s)$ of series resonant converter obtained from (21) for the values stated in table 1. The open loop gain margin (GM) is ~ 6 dB and open loop phase margin (PM) is ~ 3°. The system bandwidth of open loop system is 9.8 kHz which is sufficient for fast response, but the GM and PM is not sufficient and system may become unstable for small noise value.

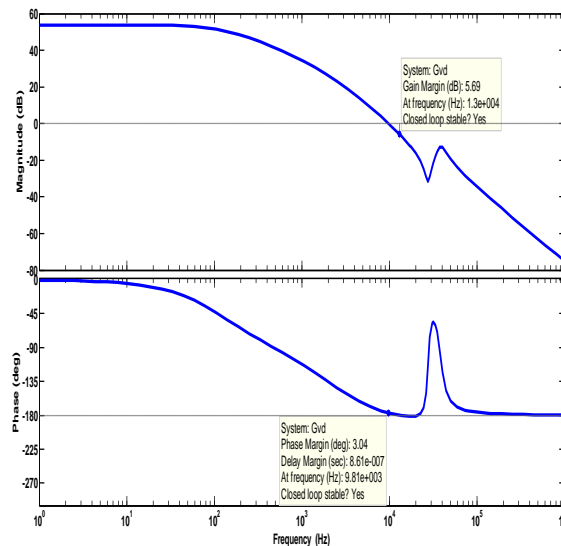


Figure 3: Open loop frequency response of series resonant converter

Simulation of closed-loop system

The feedback control circuit regulates the output DC voltage to a desired value. The output voltage should be kept constant, regardless of changes in the input voltage or load current. The duty ratio of switch is varied to control the output voltage. The error signal then controls the phase shift between two diagonal switches of the controller to keep output voltage constant. Figure 4 shows the typical architecture of a system to regulate output voltage of SMPS.

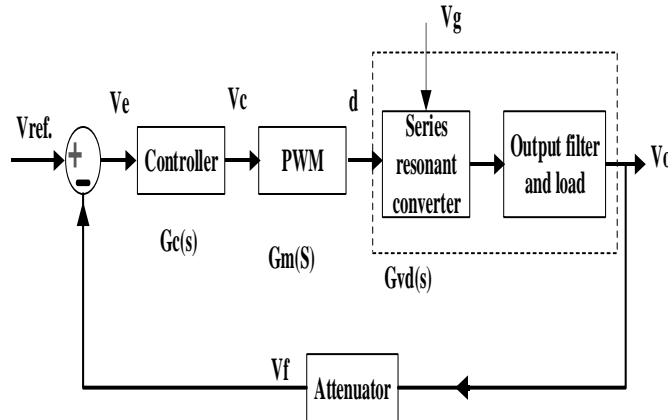


Figure 4: the control block diagram of closed-loop system

PI controller is most commonly used in SMPS, in frequency domain the transfer function of PI controller for $R_1 = 8.2 \text{ k}\Omega$, $R_2 = 91 \text{ k}\Omega$, $C = 0.1 \text{ }\mu\text{F}$ is given by

$$G_c(s) = \frac{V_c(s)}{V_e(s)} = 11 \left(\frac{1+9.1 \times 10^{-3}s}{9.1 \times 10^{-3}s} \right) \tag{22}$$

PWM converter is used to vary duty cycle of power converter switches, depending on control voltage v_c . In PWM a saw-tooth waveform of amplitude v_p , is compared with control voltage v_c , to produce a variable duty cycle, therefore the transfer function of PWM for $v_p = 2.7$ in our case is given by:

$$G_m(s) = \frac{\bar{d}(s)}{\bar{v}_c(s)} = \frac{1}{v_p} = \frac{1}{2.7} \tag{23}$$

The output of power circuit is high voltage (550 V DC) as compare to the control circuit elements operating range, hence the output is attenuated using potential divider network. The transfer function of attenuator using the value listed in table 1 is given by:

$$H_a(s) = \frac{1}{73} \tag{24}$$

The close loop transfer function is given by equation 25

$$G(s)H(s) = G_{vd}(s)G_c(s)G_m(s)H_a(s) \tag{25}$$

The value of the different transfer function is substituted in equation 25 and close loop transfer function $G(s)H(s)$ for series resonant converter is obtained using MATLAB.

$$G(s)H(s) = \frac{3.21 \times 10^5 s^4 + 3.05 \times 10^{10} s^3 + 1.02 \times 10^{16} s^2 + 7.32 \times 10^{20} s + 8.03 \times 10^{22}}{82 \times 10^{-5} s^6 + 108.2 s^5 + 5.18 \times 10^7 s^4 + 3.42 \times 10^{12} s^3 + 3.63 \times 10^{16} s^2 + 2.62 \times 10^{19} s} \tag{26}$$

The close loop response of series resonant converter is shown in figure 5. The close loop GM and PM of the system is 30.5 dB and 40.4° respectively. Simulation shows that system is stable and has bandwidth of $\sim 2.23 \text{ kHz}$ which is sufficient for fast output voltage response.

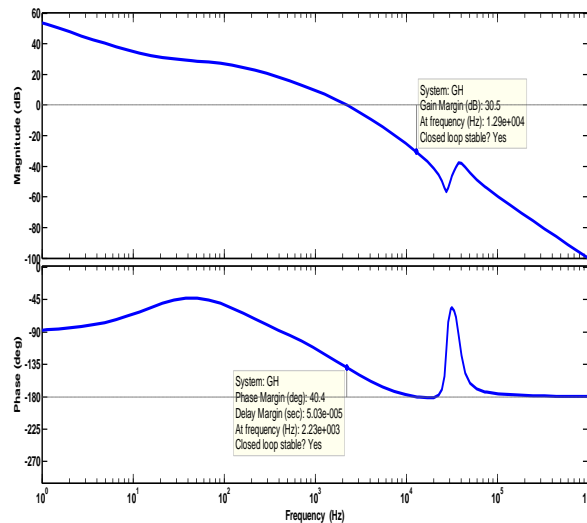


Figure 5: Closed loop response of series resonant converter

IV. RESULTS AND CONCLUSION

The small-signal modelling technique based on Generalized State Space Averaging can be applied to the Series Resonant DC/DC Converters. In this paper, the small signal model of an open loop phase-shifted full bridge series resonant converter is built and the closed loop system for this open loop system is also simulated by using MATLAB. According to the simulation, the steady-state output of the open loop system fits well with theoretical calculation, and the closed loop system can also meet the needs of disturbance resistance. Concluded from the Mathematical analysis and the simulation of power electronic circuits, Generalized StateSpace Averaging is suitable for the PWM converter with resonant circuits.

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